Clearly articulate the specific research question and the goals of the project:
250 word limit

This project aims to use numerical computations to tackle transport problems in a steady state. Either the heat transport or the elastic wave scattering directions can be pursued. The first part of the project would be to establish the equivalent of the scattering (Lippman- Schwinger) equation for the relevant problem, in terms of Green's function of free space and of the linear response coefficients of the materials, to assess how different it is from the case of electromagnetic waves. Then, we would use this equation to write a numerical solver. This powerful solver, following the logic of existing code on electromagnetic waves, would allow to exactly compute the full response of the medium to any external excitation, and therefore characterize the effect of structure on the response of the medium. Time allowing, this would be a good starting point to develop an optimization strategy: from the exact response of the medium, using automatic differentiation, we can in principle choose the positions and response coefficients of inclusions so as to craft any desired response.

Provide sufficient background to contextualize your question or problem. An educated, non-expert reader should be able to fully understand your topic. Be sure to describe the significance of this research as well: e.g. How is it unique? Why is it important? What will it contribute to the field? Include references:
600 word limit

Transport of waves and heat through many-body media is generally well understood analytically, and described numerically, in two limits: either that of periodic media like crystals, where periodicity leads to considerable simplifications of the solutions of the propagation and heat equations, or in the limit of thin samples, where transport is well described by assuming that each element of the medium interacts with transport independently. Wave and heat transport through thick aperiodic and, even worse, random media, is much more challenging to describe. In particular, it has been proposed [1- 3] that long-range correlations in the positions of small objects could lead to transmission gaps for electromagnetic waves, a hypothesis we are actively checking using Green's function approach to electromagnetic wave scattering in complex media [4].

However, the effect of such long-range correlations remains largely unexplored in the contexts of elastic waves, and of heat transport. In the former case, in most real-life applications, we can no longer ignore the anisotropy of local responses due to the tensorial nature of elastic responses. How this anisotropy couples to density-density correlations of the medium in the limit of thick media is a completely open question and could lead to exciting developments in the design of novel mechanical metamaterials, for instance, materials that do not transmit sound waves at some frequencies, negative- impedance materials, or materials that display localized responses to global excitations. In the case of heat transport, close to nothing is known about the effect of density correlations on transport properties, and one could think of optimizing a structure to be a good insulator while having a small mass density or, conversely, to be a good conductor at a minimal cost in terms of total mass.

What hinders the characterization of such systems is that most numerical work focuses on solving the full-time- or frequency-domain problem using finite elements or finite differences algorithms. However, we aim to address steady-state transmission properties in the presence of a source using a lightweight Green's function formalism, which is useful in the limit of a small volume fraction of inclusions in a homogeneous host medium.

[1] Light in correlated disordered media, K. Vynck et al., ArXiv 2106.13892 (2021)
[2] Pseudogap and Anderson localization of light in correlated disordered media, R. Monsarrat et al., Phys. Rev. Research 4(3), 033246 (2022)
[3] Wave propagation and band tails of two-dimensional disordered systems in the thermodynamic limit, M. Klatt et al., PNAS 119(52), e2213633119 (2022)
[4] High-density hyperuniform materials can be transparent, O. Leseur et al., Optica 3(7), 763 (2016)

Describe the methodological approach you will employ to carry out your proposed research:
600 word limit

The project will be a blend of guided learning and self-exploration. We seek to understand heat or sound transport through correlated disordered media, complex systems where the spatial correlation of scatterers plays a crucial role. For the learning part, we have met and discussed with the professor for learning the specific relevant physical problems, and we are reading the corresponding papers the professor assigned to gain a solid background in these physical transport problems.

Once we have enough background knowledge, our methodological approach begins by drawing parallels from the field of light transport, specifically the discrete dipole approximation method we've been learning about. We will adapt the Green's function formalism used in light scattering to solve the Lippmann-Schwinger equation for several other transport problems we are interested in. We are motivated by studies such as illustrated in this paper https://arxiv.org/abs/2005.01372, which show that the method can be efficiently applied to other transport problems, such as acoustic waves. We would then build our own numerical solvers for this certain new problem and examine the results.

One of the important parts of the numerical implementation is that we'll be trying to reduce a scattering problem into a linear system, a more computationally manageable method for steady-state solutions compared to the more complex methods like FDTD or FEM, especially given the scales we're interested in. To better illustrate this, it's about having an established powerful formalism that is used in a certain area (optics), and we are trying to find a way to adapt it to the various systems and examine the responses, such as understanding how disordered scatterers can influence transport properties in different situations. In practical terms, we will be coding Green's functions and tackling the linear problems they present, for example, designing a unique fast matrix inversion algorithm for complex symmetric linear systems that would appear in our problem to speed up the existing methods.

With the guidance from Prof. Martiniani and Dr.Mathias', this hands-on experience will be invaluable. We'll apply what we learn to investigate the effects of correlated disorder on heat or sound transport, which will be our contribution to this cutting-edge field.

Describe your timeline for completing this research, including the project's start and end dates and estimated number of hours per week dedicated to the work:
150 word limit

Our research project has already started, with an established goal to reach completion by May 2024. Over the course of this timeline, we plan to dedicate approximately 10 hours each week to our work. The initial phase will focus on literature review and familiarization with Green's function approach, estimated to take the first quarter of our timeline. Following this, we will progress into the development phase, where we will construct and refine our numerical solver, a task we anticipate will consume the bulk of our allotted time due to its complexity and the need for meticulous testing. The final phase, earmarked for the last quarter before our end date, will involve applying our solver to various scenarios, analyzing results, and optimizing our approach. This will also include time to prepare our findings for presentation and potential publication. The timeline is structured to allow for a balanced workload alongside our academic responsibilities.

How is your project relevant to your academic interests and goals?
150 word limit

As mathematics majors with a keen interest in practical applications, we are seizing this opportunity to deepen our understanding of numerical methods and computational modeling, areas that are pivotal to our intended graduate studies. Moreover, it promises to enhance our proficiency in areas like differential equations and linear algebra, which are foundational to advanced mathematical studies. By engaging in this research, we aim to build a robust portfolio of skills and experiences that will not only bolster our graduate school applications but also prepare us for the rigorous analysis and problem-solving that lie ahead.

Describe your relationship with your project mentor, addressing the following points: How did you identify your project mentor? In what capacity did you work with your mentor in developing your research project? How will you work with your faculty mentor on this project? How often will you meet?
200 word limit

We connected with our project mentor through Billy's participation in the AM-SURE program, which matches undergraduates with faculty for research experiences in applied mathematics. Moving forward, we will work under the day-to-day supervision of Mathias, a postdoctoral researcher in our mentor's lab. His role will be hands-on, providing us with immediate assistance and feedback as we navigate the complexities of our project.

We will hold formal meetings with our faculty mentor every two to three weeks to review our progress, address any challenges, and receive strategic advice. Additionally, we have established a Slack channel for ongoing communication, which will allow us to seek guidance and share updates more frequently, ensuring continuous engagement with our mentor throughout the research process. This blend of structured and flexible interaction is tailored to foster an environment of steady growth and learning.

Describe how each member will contribute to the overall project:
350 word limit

Our team comprises members with a unified goal of understanding and modeling transport problems through numerical computations, each bringing a unique set of skills and perspectives to the table.
At the outset, we will collectively delve into the foundational materials, ensuring each member has a solid understanding of Green's function formalism, the Lippmann-Schwinger equation, and the principles of correlated disordered media. These initial sessions will be crucial for establishing a common knowledge base from which we can each branch out into more specialized roles.

Once we have a shared grounding, we'll divide the coding tasks according to individual strengths and learning goals. We will regularly come together to discuss our progress, troubleshoot issues, and ensure that each member understands all parts of the code. This approach ensures that while each person is responsible for a specific aspect, we maintain a cohesive understanding of the project as a whole.
As we progress, we will continue to learn from each other. When one member encounters a new challenge or discovers an efficient solution, they will share this with the group, contributing to the collective expertise. Regular meetings will be held to review code, discuss theoretical developments, and plan subsequent steps.

If a member completes their task ahead of schedule, they will assist others, balancing the workload and ensuring that no one falls behind. This collaborative spirit extends to our problem-solving approach: challenges will be tackled as a group, ensuring that all members have a say in the direction and methodology of the project.